THE PLANIMETER

INSTRUCTIONS

to measure areas & volumes

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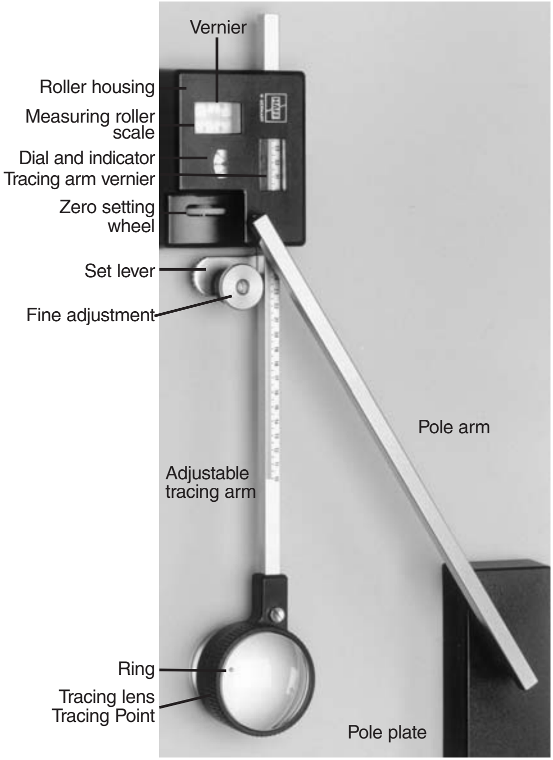
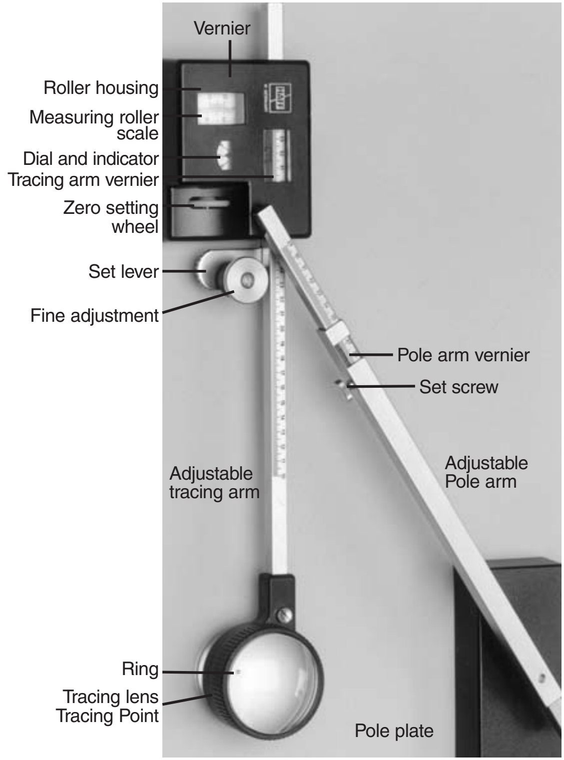
HAFF-Planimeter No. 313 (for cm²)
(Ordering No. 313E) only in case



HAFF-Planimeter No. 315 (for cm² and sq.in.)
(Ordering No. 315E) only in case



HAFF-Planimeter No. 317 (for cm^2)
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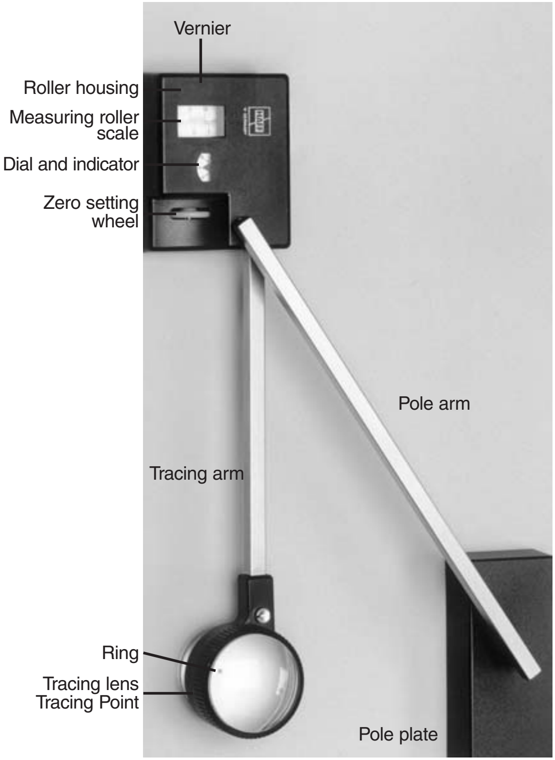


Although these instructions are written on the assumption that the reader has little or no knowledge of planimeters, engineers may well find them to be of interest, particularly towards the end of the booklet.

HAFF Planimeters are fitted with a large, crystal-clear tracing lens, and a setting wheel with which to zero the scales. The lens enlarges the view of the line being traced and so makes possible a very high degree of accuracy. The setting wheel is used to zero the scales without touching the internal mechanism.

A. Introduction

The planimeter is a simple instrument for the precise measurement of areas of plane figures of any shape. To measure an area it is only necessary to trace the outline of the figure in a clockwise direction with the centrepoint (within the ring) of the tracing lens and to read off the result on the scales.

The planimeter consists of 3 separate parts; the tracing arm to which is attached the roller housing the pole arm and the pole plate. The three parts are packed separately in the case. The pole arm is a simple beam. On each end is fixed a ball, one for fitting into the roller housing, the other into the pole plate. The roller housing rests on three supports; the tracing lens, the measuring roller and a supporting ball.

B. Important points

Haff Planimeters are manufactured to give many years of accurate, trouble-free service but as with all precision instruments they must be handled carefully. Whenever the planimeter is not in use it should be stored safely in its case.

The most easily damaged parts are the rim and bearings of the roller. The measuring roller is made of hardened steel and has a milled edge. Always use the setting wheel to zero the scales.

The roller should run freely with only a little end-float. The accuracy of the planimeter can be checked at any time with the aid of the test area which is provided (see Section D 1).

C. Using the planimeter

Reading examples

Set up the planimeter so that the tracing arm and lens are towards you. Attach the pole arm to the roller housing and to the pole plate. The pole arm should be approximately at right angles to the tracing arm, with the pole on the right (Fig.1).

First move the zero setting wheel and watch the scale and the dial. Stop as soon as the 0 on the dial is covered by the indicator and the 0 on the scale is opposite the 0 on the vernier. The instrument is now in its zero position (Fig. 2).

Now move the tracing lens very slightly to the right and stop before the 0 on the vernier has reached the first of the calibrations on the scale. If now, for example the 4th calibration on the vernier matches a calibration on the scale, then the scale has moved four Vernier Units (VU) towards the first calibration (Fig. 3).

Next move the tracing lens a little further to the right until the 0 on the vernier is opposite the first calibration on the scale. The scale has now moved 10 vernier units (10 VU) or one calibration (Fig. 4).

If the scale rotates until the 1 on it is opposite the 0 on the vernier, it has then turned through 100 vernier units (100 VU) or ten calibrations (Fig. 5).

When the scale has made a complete rotation (passing all the figures from 1 to 9) and has returned to 0, it has turned through 1000 vernier units (1000 VU) or 100 calibrations. The dial now indicates 1 instead of 0 (Fig. 6). Each of the ten figures on the dial corresponds to a complete revolution of the scale – 1000 VU.

To count the total number of units, we read the thousands on the dial, the hundreds and tens on the scale, and the units on the vernier.

A final example should make the method of reading quite clear. In Fig. 7 the dial is between 3 and 4, so the answer is between 3000 and 4000 VU, the scale has a reading between 47 and 48, so the answer is between 3470 and 3480 VU and the fourth calibration on the vernier corresponds with a calibration on the scale, giving an answer of 3474 VU.

This is the sequence of observations which must be made whenever a planimeter reading is taken, and each of the four figures must be checked carefully. With a little practice the readings can be taken quickly and without error.

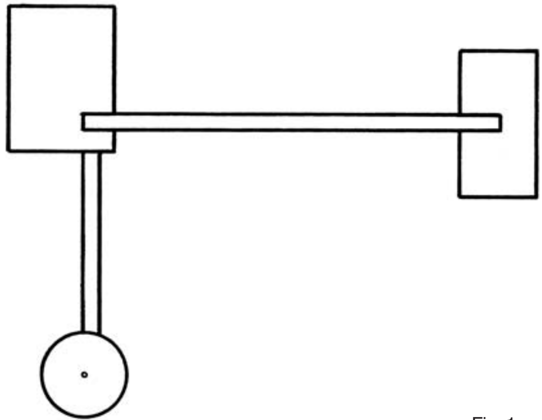


Fig. 1

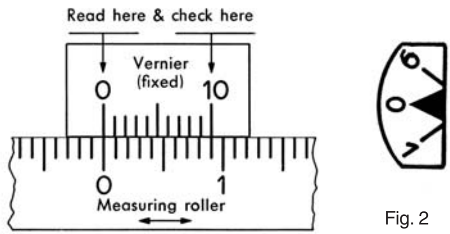


Fig. 2

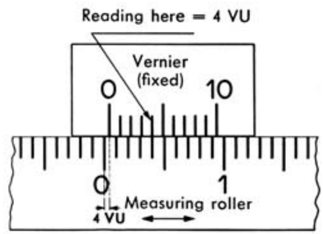


Fig. 3

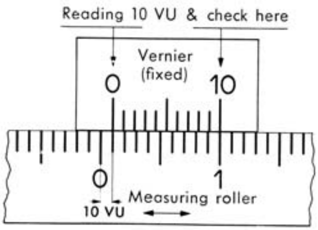


Fig. 4

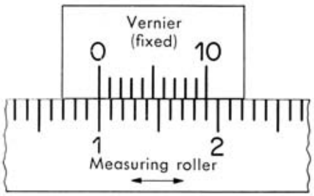


Fig. 5

Fig. 6

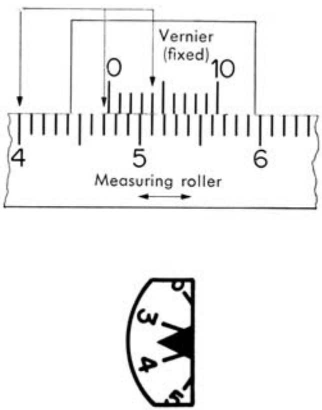


Fig. 7

The value of the vernier unit (VU)

The HAFF Planimeter No. 317 has a fixed tracing arm which is set so that the value of the vernier unit is always 0.1 sq.cm.

		Number of VU	sq.cm
One vernier unit (VU)	=	1	0.1 sq.cm
One calibration on the scale	=	10	1.0 sq.cm
Distance between the numbers on the scale	=	100	10.0 sq.cm
One revolution of the scale or one calibration on the dial	=	1000	100.0 sq.cm
One revolution on the dial	=	10,000	1000.0 sq.cm

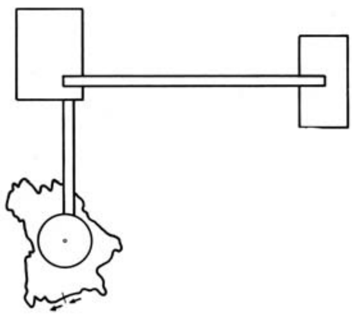
The HAFF Planimeters No. 313 and 315 have adjustable tracing arms, the length of which can be varied to select the most useful vernier unit value shown in Table III and IV (page 15).

Measuring with

"Pole plate outside the figure" (usual)
For these working areas:

No.	313			315			317		
cm	20 x 40	5 x 65	30 Ø	20 x 40	5 x 84	30 Ø	20 x 40	5 x 65	30 Ø

The pole plate may be set down in any position outside the figure which allows the tracing lens to be guided round the entire outline. In the case of larger figures consult Section E2. The length of the pole arm has no effect on the measurement when the pole plate is set down outside the figure.



Starting and finishing point

Fig. 8

Tracing an area

1. Before tracing an area the tracing lens may be set down in the middle of the area. The pole arm should be approximately at right angles to the tracing arm. Mark the starting point (which will also be the finishing point) with a line at right angles to the outline (Fig. 8).

2. Position the small centre ring of the tracing lens (which should be held in the right hand) exactly over the starting point (Fig. 9). At the same time, with the left hand, turn the zero setting wheel until the dial and the scale both return to zero (Fig. 10).

3. Holding the tracing lens as shown in Fig. 9, trace the outline in a clockwise direction with the small centre ring. Keep looking in the direction in which the lens is to travel and try to keep the line inside the ring.

4. It is impossible to hold the small ring exactly over the centre of the line all the time, so compensate for errors caused by going off the line to one side by going off an equal amount in the opposite direction. Extensive trials show that these errors do balance out.

5. When the outline has been traced and the small ring has returned to the starting point, the reading is taken. Let us assume that the reading is 4175 Vernier Units (VU).

6. The figure 4175 is the number of vernier units in the area. If the HAFF Planimeter No. 317 is being used, the value of the Vernier Units is 0.1 sq.cm and the area is 4175 x 0.1sq. cm. If the HAFF Planimeter No. 313 or 315 is being used, the value of the vernier unit and therefore the area, will depend upon the setting of the tracing arm. To find this setting, consult the table in the case and Table III or IV page 15.

7. It is good practice to retrace the perimeter in order to check the accuracy of the measurement. The accuracy can be increased by taking the average of several readings – see Section D on the subject of accuracy.

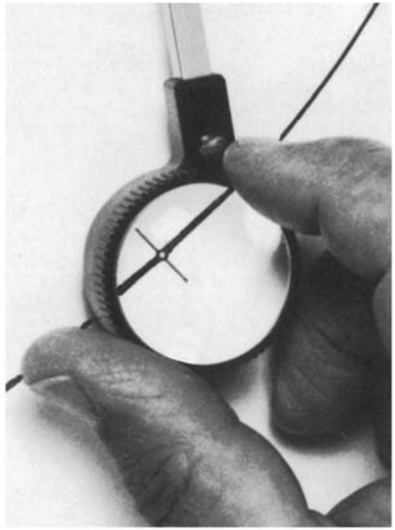


Fig. 9

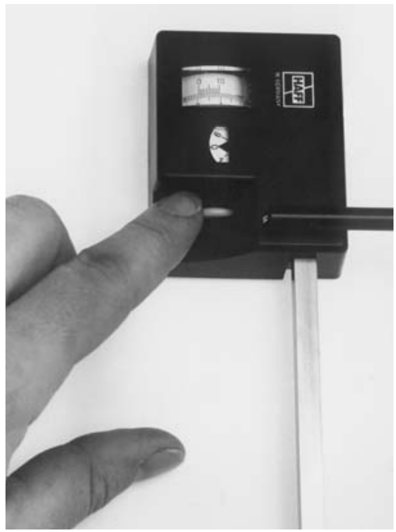


Fig.10

Areas in different scales

The planimeters Nos. 313, 315 and 317 can be used not only to measure areas in square inches or square centimetres in a scale of 1:1, but equally well to measure areas in any scale, providing that the measurements in sq.in. or sq.cm are converted with the aid of the appropriate factor. Shown in Tables I and II (pages 13 and 14).

Example – Planimeter No. 317 (m-system)

Assume that a map is drawn to a scale of 1:5000. If an area measured on the map is 125.3 sq.cm, the actual area is 125.3 x factor 2500 = 313.250 sq.m or 125.3 x factor 0,25 = 31.325 ha (see Table II).

Example – Planimeter No. 313 or 315 (m- and inch-system)

If we assume that the area mentioned in the above examples has been measured with the Planimeter No. 313 or 315, the same result can be obtained if the tracing arm has been set according to the table in the case so that 1 Vernier Unit is equal to 0.01 sq.in. (0.1 sq.cm). It is better however to aim at greater accuracy, and this can be achieved by choosing a shorter tracing arm setting as shown in Tables III and IV. This makes the value of the vernier unit smaller; for example, if we make the value 0.008 sq.in. (0.04 sq.cm), the measurement may now be 1346 (3130) Vernier Units or 1346 x 0.008 = 10.912 sq.in. (3130 x 0.04 = 125.20 sq.cm), which gives a full size area of 10912 sq.in. x factor 0.0062 = 67.65 sq.miles. (125.20 sq.cm x factor 0,25 = 31.30 ha.) Table I and II.

If an area is to be measured in a scale which is not shown in Table I or II, the necessary factor, by which the area in sq.in. or sq.cm must be multiplied in order to obtain the result in sq.ft. or sq.m, can be calculated as follows:

Inch

$$f = \left(\frac{n}{12}\right)^2$$

Metric

$$f = \left(\frac{n}{100}\right)^2$$

Where f is the required factor and n is the scale relationship.

Example (Inch):

The factor f is required for a map drawn to a scale of 1/5 in. to 1 ft. i. e. 1:60.

$$f = \left(\frac{60}{12}\right)^2 = 25$$

The area on the map measured in sq.in. can be expressed in sq.ft. by multiplying it by 25.

Example (Metric):

The factor f is required for a map drawn to a scale of 1:1820.

$$f = \left(\frac{1820}{100}\right)^2 = 18{,}20^2 = 331{,}24$$

The area on the map measured in sq.cm can be expressed in m^2 by multiplying it by 331.24.

Tracer-arm settings for usual scales

arm setting	English scale	Area for 1 vernier unit	Area for 1 revolution
20.27	1:1	0.01 sq.in.	10 sq.in.
20.27	1:1200	100 sq.ft.	100000 sq.ft.
25.34	1:2400	500 sq.ft.	500000 sq.ft.
12.67	1:4800	1000 sq.ft.	1000000 sq.ft.

arm setting	Meter scale	Area for 1 VU	Area for 1 revolution
31.41	1:1	0.1 cm^2	100 cm^2
31.41	1:1000	10 m^2	10000 m^2
25.13	1:2500	50 m^2	20000 m^2
12.57	1:5000	100 m^2	100000 m^2

Table I (Inch-System)

Scale shown on the map or plan	Scale	Factors by which the measured area in **sq. in.** must be multiplied to give the answer in the units shown at the head of each of the columns.			
	1 : n	sq. in.	sq. ft.	acres	sq. miles
Full Size	1 : 1	1.0	—	—	—
½ Full Size	1 : 2	4.0	—	—	—
¼ Full Size	1 : 4	16.	—	—	—
3 in. = 1 ft.	1 : 4	16.	0.111	—	—
1½ in. = 1 ft.	1 : 8	64.	0.444	—	—
1 in. = 1 ft.	1 : 12	144.	1.0	—	—
¾ in. = 1 ft.	1 : 16	256.	1.778	—	—
½ in. = 1 ft.	1 : 24	576.	4.0	—	—
⅜ in. = 1 ft.	1 : 32	1024.	7.11	—	—
¼ in. = 1 ft.	1 : 48	2304.	16.0	—	—
⅛ in. = 1 ft.	1 : 96	9216.	64.0	—	—
1 in. = 10 ft.	1 : 120	—	100.	—	—
1 in. = 20 ft.	1 : 240	—	400.	—	—
1 in. = 40 ft.	1 : 480	—	1600.	0.0367	—
1 in. = 50 ft.	1 : 600	—	2500.	0,0574	—
1 in. = 66 ft.	1 : 792	—	4356.	0.1	—
1 in. = 80 ft.	1 : 960	—	6400.	0.147	—
1 in. = 100 ft.	1 : 1200	—	10000.	0.2296	—
—	1 : 1250	—	10851.	0.2491	—
1 in. = 132 ft.	1 : 1584	—	17424.	0.4	—
1 in. = 166.66 ft.	1 : 2000	—	27777.	0.6377	—
1 in. = 200 ft.	1 : 2400	—	40000.	0.918	—
—	1 : 2500	—	43403.	0.9963	—
1 in. = 300 ft.	1 : 3600	—	90000.	2.0664	—
1 in. = 330 ft.	1 : 3960	—	108900.	2.5	—
1 in. = 400 ft.	1 : 4800	—	160000.	3.673	—
—	1 : 5000	—	173611.	3.9853	0.0062
1 in. = 660 ft.	1 : 7920	—	435600.	10.0	0.0156
1 in. = ¼ mi.	1 : 15840	—	—	40.0	0.0625
—	1 : 25000	—	—	99.6	0.1557
1 in. = ½ mi.	1 : 31680	—	—	160.0	0.25
—	1 : 62500	—	—	622.7	0.973
1 in. = 1 mi.	1 : 63360	—	—	640.0	1.0

Table II (Metric System)

Scale	Factors by which the measured area in **sq. cm** must be multiplied to give the answer in the units shown at the head of the columns.			
1 : n	sq. cm	sq. m	ha	sq. km
1 : 1	1.0	—	—	—
1 : 1*	0.155	—	—	—
1 : 2	4.0	—	—	—
1 : 2.5	6.25	—	—	—
1 : 5	25.0	—	—	—
1 : 10	100.0	0.01	—	—
1 : 15	225.0	0.0225	—	—
1 : 20	400.0	0.04	—	—
1 : 25	625.0	0.0625	—	—
1 : 30	900.0	0.09	—	—
1 : 33⅓	—	0.111	—	—
1 : 40	—	0.16	—	—
1 : 50	—	0.25	—	—
1 : 75	—	0.5625	—	—
1 : 100	—	1.0	—	—
1 : 125	—	1.5625	—	—
1 : 250	—	6.25	—	—
1 : 500	—	25.0	—	—
1 : 650	—	42.25	—	—
1 : 750	—	56.25	—	—
1 : 1000	—	100.0	0.01	—
1 : 1250	—	156.25	0.0156	—
1 : 1440	—	207.36	0.0207	—
1 : 1500	—	225.0	0.0225	—
1 : 2000	—	400.0	0.0400	—
1 : 2500	—	625.0	0.0625	—
1 : 2880	—	829.44	0.0829	—
1 : 3000	—	900.0	0.09	—
1 : 4000	—	1600.0	0.16	—
1 : 5000	—	2500.0	0.25	—
1 : 10000	—	10000.0	1.0	0.01
1 · 25000	—	—	6.25	0.0625
1 : 50000	—	—	25.0	0.25
1 : 100000	—	—	100.0	1.0

* For measuring areas in the inch system with the planimeter 317. The factor 0.155 is sq. in.

Table III (Inch-System) for 313 and 315

Tracing arm setting (Adjustable with the 10 sq. cm Test Area)		Correct no. of vernier units in 10 sq. in. Test Area	Area
1	2	3	4
approximate	±..........% exact	exact	exact
20.27		0.010 sq. in.	10 sq. in.
32.43		0.016 sq. in.	10 sq. in.
28.38		0.014 sq. in.	10 sq. in.
24.33		0.012 sq. in.	10 sq. in.
16.22		0.008 sq. in.	10 sq. in.
12.16		0.006 sq. in.	10 sq. in.

Table IV (Metric-System) for 313 and 315

Tracing arm setting (adjustable with the 10 sq. cm Test Area)		Correct no. of vernier units in 100 sq. cm Test Area	Area
1	2	3	4
approximate	±...........% exact	exact	exact
31,41		0,1 sq. cm	100 sq. cm
28,27		0,09 sq. cm	100 sq. cm
25,13		0,08 sq. cm	100 sq. cm
21,99		0,07 sq. cm	100 sq. cm
18,85		0,06 sq. cm	100 sq. cm
15,71		0,05 sq. cm	100 sq. cm
12,57		0,04 sq. cm	100 sq. cm

Adjustment of the setting of the tracing arm for variations in paper (313 and 315)

It is possible that measurements made on some papers may give results which are slightly incorrect. This error can be removed by making a very small change in the setting of the tracing arm.

Tables III and IV show in column 1 the approximate tracing arm settings for various vernier values. They are calibrated at the factory. Column 2 is provided so that the exact values may be recorded for the individual instrument after they have been determined with the aid of the test area (Fig.11) for 100 sq.cm.

The procedure is as follows:
Position the tracing arm see Section E in accordance with the label in the case so that, for example, at a setting of 31.41, 1 vernier unit (VU) = 0.1 sq.cm and 1 revolution of the measuring roller scale = 100 sq.cm when the scale is 1:1.

Trace round the test area once. Make a note of the resulting reading in VU. If this is not exactly 1000, adjust the tracing arm setting to correct the error. (When the tracing arm setting is reduced, the reading will be increased, and vice versa.) When repeated measurements with the test area have proved the accuracy of this setting, read the position of the tracing arm, for example 31.54. This new setting is 1.004 times the original setting of 31.41.

Now, all the readings given in column 1 of Table IV should be multiplied by 1.004 and written in column 2. (Also in Table III nevertheless the test for 100 sq.cm was used).

Also, all the tracing arm settings on the label in the case should be multiplied by 1.004 before starting to take measurements.
This instruction is equally true for both the inch, and metric type planimeters.

D. Accuracy

1. The precision of the instrument

The accuracy of the instrument can easily be checked with the added test area (ellipse) for 100 sq.cm.

This area is in the form of an ellipse having an area of exactly 100 cm^2 printed on film (Fig.11).

a) Place the Planimeter as shown in the illustration, and measure the area of the ellipse as described on page 9.

   The measurement is correct when the measuring roller has made one complete revolution – from 0 to 0. This is equal to 1000 vernier units x 0.1 = 100 cm^2. See Figs. 2 and 6.

b) You can make some more circuits and record the reading after each. Two characteristics sets of readings are given below.

Check A	Check B
1001 VU	1000 VU
2002 VU	2000 VU
3002 VU	2999 VU
4002 VU	3999 VU
5001 VU	4999 VU
6001 VU	5998 VU
7002 VU	6997 VU
8002 VU	7997 VU
9003 VU	8996 VU
10003 VU	9996 VU

Check A shows a small error in the initial setting of the vernier. The readings show how accurate the planimeter is. The error present in the first reading remains about the same in each successive reading, thus we can justifiably attribute it to human error in the initial setting.

Check B shows a small but increasing deviation as each measurement is made.

Both checks indicate an instrument error of less than 0.1 % when measuring an area of 100 sq.cm (10 sq.in.).

c) Further checks can be made with various distances between the pole and the test area, moving the pole from the right to the left side, and by tracing in the opposite direction. These checks will show small variations in the results which vary for each instrument. The main purpose is to ensure that the instrument is in a generally good condition and is capable of giving consistent results. If it is found that the instrument gives higher readings when the pole is on one side rather than the other, a measurement can be taken with the pole in each position and the average used.

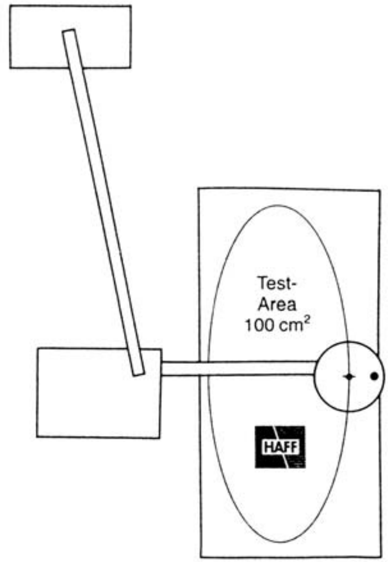


Fig. 11

2. Environmental factors

External conditions are just as important as the accuracy of the individual instrument. The quality of the surface over which the measuring roller moves is perhaps the most important factor. If the paper is crumpled or torn, or has pin-holes in it, or if it is wavy or uneven in any way, accurate results can not be expected. The texture of the paper does not matter as long as it is constant over the whole surface. When working on an inclined plane the accuracy will be reduced and the possibility of accidental damage increased. The instrument is accurate at 20 degrees Centigrade = 68 degrees Fahrenheit.

3. Human error

Some people obtain better results than others. A good eye, a steady hand and patience when tracing the outline are most important for good results. Good light and a comfortable working position also play a decisive role.

4. The size of the area to be measured

If a square with sides 1 in. long is traced with an error of 0.01 in. outside the square, the area will be shown as 1.04 sq.in. instead of 1 sq.in. = a 4 % error. If the same mistake is made with a 2 in. square, the result will be 4.08 sq.in. = a 2 % error. The result for a 10 in. square will be 100.4 sq.in. = a 0.4 % error. The larger the figure, the smaller the percentage error.

Areas larger than that shown in Fig. 3 can be measured by placing the pole inside the figure. The accuracy of the measurements made in this way can be checked by drawing large circles of known radius and comparing the measured area with the calculated area. The values agree very closely, and it cannot be said that measurements taken with the pole inside the figure are, to any significant extent, less accurate than those taken with the pole outside. It has also been found that, contrary to statements often made in instruction books for planimeters, the accuracy does not decrease when the area is close to that of the neutral circle.

To sum up, we can say, that repeated careful measurements in favourable conditions will ensure an accuracy within plus or minus 0.1 % with HAFF Planimeters, when the area to be measured is greater than 100 sq.cm (10 sq.in.).

E. Significancy of the adjustable tracing arm and pole arm of 313 and 315

1. The adjustable tracing arm

a) Purpose

As shown in the Tables III and IV several Vernier Units (VU) can be choiced. Especially for measuring small areas, small Vernier Units should be prefered. The accuracy of the result will be better.

Every planimeter has its own characteristics which change in the course of time as parts wear. These changes can be compensated for, by adjustment to the tracing arm setting. Any change in the scale of old maps – caused perhaps by instability in the material on which they were drawn - can be allowed for in this way. A check can be made at any time with the test area, and any necessary correction can be made by the user himself – see Section D. (In the case of the No. 317 Planimeter a check can be made but there is no provision for adjustment).

b) Setting the arm

Loosen the set lever by moving the lever to the right. The roller housing can now be slid along the tracing arm, and using the fine adjustment wheel, set exactly to the required reading. The procedure is as follows: - if the tracing arm is to be set to a reading shown in Tables III or IV, move the roller housing until the zero of the tracing arm vernier is close to the required setting, then with the thumb of the right hand press the fine-adjustment roller against the tracing arm. Light pressure is then sufficient to move the roller housing to the right or left until the setting is exactly as required. The set lever is moved to the left to lock it and the setting is checked. (The general principle of using the vernier is set out in Section C, but in this instance be sure to read the scale and vernier from right to left).

The scale on the tracing arm is arbitrary and serves only to make it easy to repeat settings previously determined.

2. The adjustable pole arm on 315

a) Purpose
Measuring large areas

The length of the pole arm has no effect on the measurement when the pole is set down outside the figure, but it is advantage that with it the maximum working area can be increased. It can be useful to shorten the pole arm when working on a small drawing board.

It is completely different when measuring with the pole inside the figure. In this case the length of the pole arm is of decisive importance. When measuring areas larger than 20 x 40 cm (8 x 16 in.) or 30 cm Ø (12 in. Ø) the pole must be placed inside the figure. The circumscription is done in the same way as when the pole is outside the figure, that is, in a clockwise direction. To interpret the result, a constant which is printed on the table in the case, must be used. This constant is given in sq.cm and sq.in. and is the area of the circle which the tracing lens follows round the pole when the axis of the measuring roller is a tangent to the circle and the measuring roller does not rotate. If the area to be measured is larger or smaller than the constant circle, the reading given will be the number of vernier units by which the area is larger or smaller than the constant circle. If the area is larger than the constant circle, the scale will have moved forward, and the area according to the scale is simply added to the area of the constant circle. If the area is smaller, the scale will have moved back, so that the reading will be the complement of the number required. To calculate this number, subtract the reading from 10000. This gives the number of vernier units the scale has turned through. Change this into an area in the appropriate units and subtract it from the area of the constant circle.

The table inside the case shows the settings for both of the arms at which the constant circle has an area of 2000 sq.cm and 200 sq.in.

b) Setting the arm

Loosen the set screw by turning it to the left. The inner square beam of the pole arm can now be moved gently. The set screw is then tightened by turning it to the right.

Example 1. An area is measured with the pole inside the figure and the tracing and pole arms have been set according to the table. The reading is 2785 VU x 0.01 = 27.85 sq.in. When taking the measurement, the scale was moving forward, so the area is bigger than that of the constant circle and must therefore be added to it. 200 sq.in. plus 27.85 sq.in. is 227.85 sq.in. The area measured is 227.85 sq.in.

Example 2. (Uncommon). When taking the measurement, the scale moved backwards. The reading went to 7865 so the scale had turned through 10000 minus 7865 vernier units or 2135 VU. 21.35 x 0.01 sq.in. = 21.35 sq.in. This area is subtracted from that of the constant circle. 200 sq.in. minus 21.35 sq.in. is 178.65 sq.in. is 178.65 sq.in. = the area measured.

Planimeters No. 313 and 317

These planimeters can also be used for taking measurements with the pole inside the figure, even though they do not have adjustable pole arms. The only difficulty is that the area of the constant circle will not be a simple figure (2000 sq.cm and 200 sq.in.) as with the 315 – it is on the label in the case - but it is still used in exactly the same way.

General

An other way to measure large areas is to divide the area into measurable segments, measuring each separately and adding the results.

F. Examples of applications

To find the volume of earth required for an embankment.

Sections of the embankment are drawn on the plan, their areas are measured and plotted in the form of a graph, the area under the graph is measured and from this the volume of the embankment is calculated.

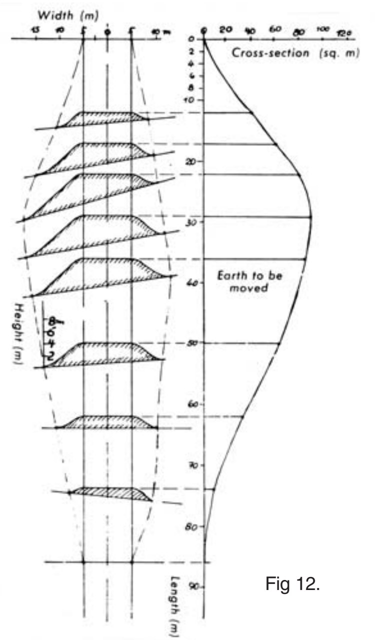


Fig 12.

The procedure is as follows.

1. From a contour map of the area, draw a series of sections as shown in Fig. 12 – the number and spacing of the sections depends on the rate of change of the shape of the sections.

   It is not necessary to have the vertical and horizontal axes drawn to the same scale, in the example shown (reduced in scale) the width of the road is in a scale of 1 cm to 5 m, while the height of the piled up earth and the length are to a scale of 1 cm to 4 m. It must be remembered however, when measuring the cross-sectional areas, that 1 cm x 1 cm represents 5 m x 4 m.

2. Measure the cross-sectional areas and transfer them to the graph drawn alongside, in their appropriate positions. Join these points with a smooth curve in the usual way.

3. The area under the graph represents a volume in which 1 cm x 1 cm x 1 cm has a value of 5 m x 4 m x 4 m or 80 cu.m. That is, the area of 1 sq.cm under the graph represents a volume of 80 cubic metres of earth.

4. This area is now traced round and, for example, a result of 57.50 sq.cm obtained. The volume represented by this value is 57.50 x 80 cu.m or 4600 cu.m.

The method is equally valid, no matter what scales or units are being used.

Calculation of the weight of a lamina

The problem is to find the weight of a lamina such as that shown in Fig. 13. First calculate the volume by multiplying the measured surface area by the known thickness of the material, then multiply this by the density. In the example shown, the area is 8.5 sq.cm the thickness 0.12 cm and the density of the material 7.86 gm/cc.
The weight is then. -

8.5 x 0.12 x 7.86 = 8.017 gm.

Fig. 13

Calculation of the capacity of a reservoir

A map showing contour lines for the area of the reservoir is required, and on this should be marked the proposed water level (Fig. 14). Then proceed as follows: -

1. In some convenient scale draw a vertical axis representing the depth of the water in the units being used. In this case the contour lines are at 10 metre intervals and the maximum depth is a little over 60 metres.

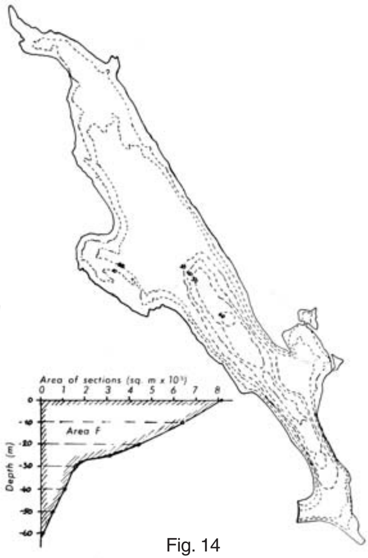


Fig. 14

2. Measure the surface area of the water and transfer this to the horizontal axis at the zero depth level. This will be the maximum area and therefore the longest horizontal measurement on the graph. Continue this process, measuring the area at each level and plotting the value on the graph. Join the points with a smooth curve.

3. Calculate the volume represented by 1 sq.cm under the curve. In the example, 1 sq.cm = 10 m depth x 1000 sq.m area = 10000 cu.m, so that 1 sq.cm under the curve represents 1×10^4 cu. of water in the reservoir.

4. Measure the area under the curve in sq.cm – in this case 18.2 sq.cm – and convert this to a volume in cu.m by multiplying by 10^4. The capacity of the reservoir is: 18.2×10^4 $= 1.82 \times 10^5$ cu.m.

We can calculate the volumes of lakes etc. in the same way (provided that the contour lines are known for the various depths) and of objects above ground level in reverse profile, so to speak, such as slag heaps, tips and mounds of all types, whether man-made or natural.

To construct the integral curve of a given curve

Erect ordinates to divide the given curve (Fig. 15), into segments. They are shown equally spaced, but it is often better to arrange the spacing to suit changes of curvature. Draw up a table with four rows and as many columns as there are ordinates – including the extremes. Starting at the right hand or upper limit, with the planimeter reading zero, trace along the x-axis towards the left, nothing in Row 2 in the table, the reading at the foot of each ordinate, until the left hand or lower limit is reached – the readings should be entered in the table in the same order as they are taken, i. e. from right to left. Having noted the last reading in the last space Row 2, put the same figure in the first – left hand – space in Row 1. Now trace round the curve as far as the intersection of the first ordinate with the curve, and then down the ordinate to the x-axis, where another reading is taken and entered in the table in Row 1 – starting from the left. Retrace the ordinate up to the curve again, along the curve and down the next ordinate to the x-axis and note the reading. Repeat the process all the way round the curve until the right hand limit is reached. Subtract the values in Row 2 from the corresponding ones in Row 1 and enter the results in Row 3. These are the integrals, expressed in vernier units, of the curve between the lower limit and each of the ordinates. Convert these into units of area by multiplying them by the appropriate factor and enter them in the last row in the table. From these, and from the known upper and lower limits of the integration, draw the graph of the integral curve (Fig. 16).

The second and succeeding integral curves can be drawn in the same way.

A-point-at-a-time integration with Planimeter 313, 315 or 317

Instructions :

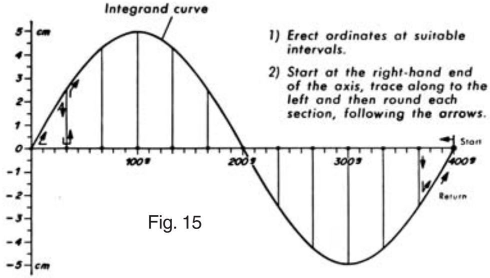


Fig. 15

3) Enter the readings in the table.

4) Enter the differences = VU.

5) Multiply by 0.1 for sq. cm.

Readings

	8628	8714	8823	8948	9075	9189	9276	9345	9398	9466	9575	9746	Return 9998
	8628	8693	8747	8794	8840	8895	8961	9049	9159	9306	9494	9727	Start 0
VU	0	21	76	154	235	294	315	296	239	160	81	19	-2
sq. cm	0	2,10	7,60	15,40	23,50	29,40	31,50	29,60	23,90	16,00	8,10	1,90	-0,2

6) Plot the integral points.

7) Draw the integral curve.

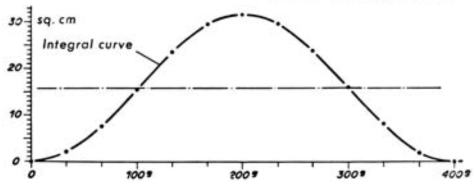


Fig. 16

G. Formulas

1.) F = L x U {sq.cm}
2.) F = N x f_o {sq.cm}
3.) L x U = N x f_o {sq.cm}

Meanings:

F = Area {sq.cm}

L = Length of the tracing arm (cm) from middle of the ring in the tracing lens up to the middle of the hole in the roller housing. (Numbers engraved on the adjustable tracing arm are L:5).

U = Circumference of the measuring roller d x π

N = Number of Vernier Units (VU) for one revolution of the measuring roller = 1000.

f_o = Value of 1 VU for scale 1:1 sq. cm or sq.in.

4.) $f = \left(\frac{n}{100}\right)^2 \times f_0$ [sq. m]

Meanings:

f = Value of 1 VU for scale 1: n sq.m

n = scale

f_o = Value of 1 VU for scale 1:1 sq.cm

5.) $f = \left(\frac{n}{12}\right)^2 \times f_0$ [sq. ft.]

Meanings:

f = Value of 1 VU for scale 1: n sq.ft.

n = scale

f_o = Value of 1 VU for scale 1:1 sq.in.

Contents

Manufacturer of
HAFF Planimeters:

GEBRUEDER HAFF GMBH
D-87459 PFRONTEN (GERMANY)

HAFF Planimeters are imported by